Numerical Study of Paramagnetic Elliptical Microparticles in Curved Channels and Uniform Magnetic Fields

Christopher Sobecki , Jie Zhang, and Cheng Wang
Microscale Transport Laboratory, Department of Mechanical and Aerospace Engineering

Background

Using magnetic fields to separate magnetic particles by shape is essential for cell separation for disease diagnosis. In previous cases, researchers separated particles in straight channels under uniform magnetic fields. Other researchers have placed spherical particles in different curved channel designs and studied their migration using inertial effects.

Objective

To numerically evaluate the time-dependent migration and rotation of a paramagnetic elliptical particle in a low Reynolds number ($R_e \ll 1$) Poiseuille flow, under a uniform magnetic field, and in a curved channel.

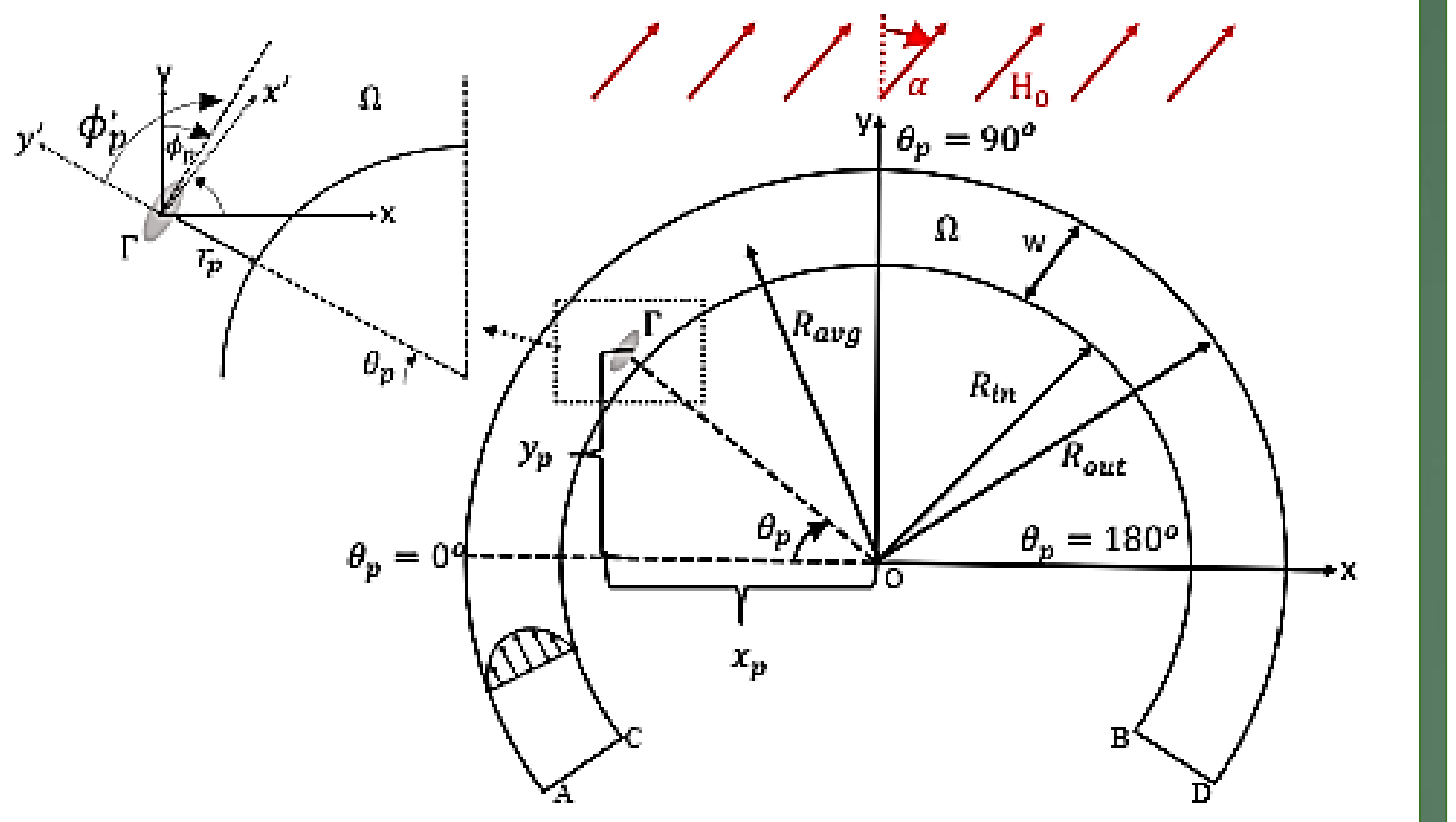

Fig1. Schematic view of the numerical model of an elliptical paramagnetic particle in a Poiseuille flow and under a uniform magnetic field with strength H_0 at direction α. The symbols Γ and Ω represents the domain elements around the particle surface and the fluid domain, respectively. The variable r_p represents the radial distance between the particle and the wall whereas θ_p represents the particle position in the curved channel.

Equations

- Global Frame of the Particle Orientation:

$$\phi'_p = \phi_p - \theta_p + 90^\circ$$

- Flow Field Using the Continuity and the N-S Equations:

$$\nabla\cdot\mathbf{u} = 0$$

$$\rho_f\left[\frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right] = -\nabla p + \nabla\cdot\nu_f(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$$

- Maxwell Equations for a Uniform Magnetic Field:

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{H} = 0$$

Material Properties

- Particle and Fluid Density:

$$\rho_p = 1100 \text{ kg/m}^3 \qquad \rho_f = 1000 \text{ kg/m}^3$$

- Fluid Viscosity: $\eta_f = 1.002\times10^{-3}$ Pa·s
- Inlet Flow Velocity: $U_{avg} = 2.5$ mm/s
- Reynolds Number: $R_e = 0.125$
- Aspect Ratio: 4
- Magnetic Susceptibility: $\chi_p = 0.26 \quad \chi_f = 0$

$\alpha = 0^\circ$

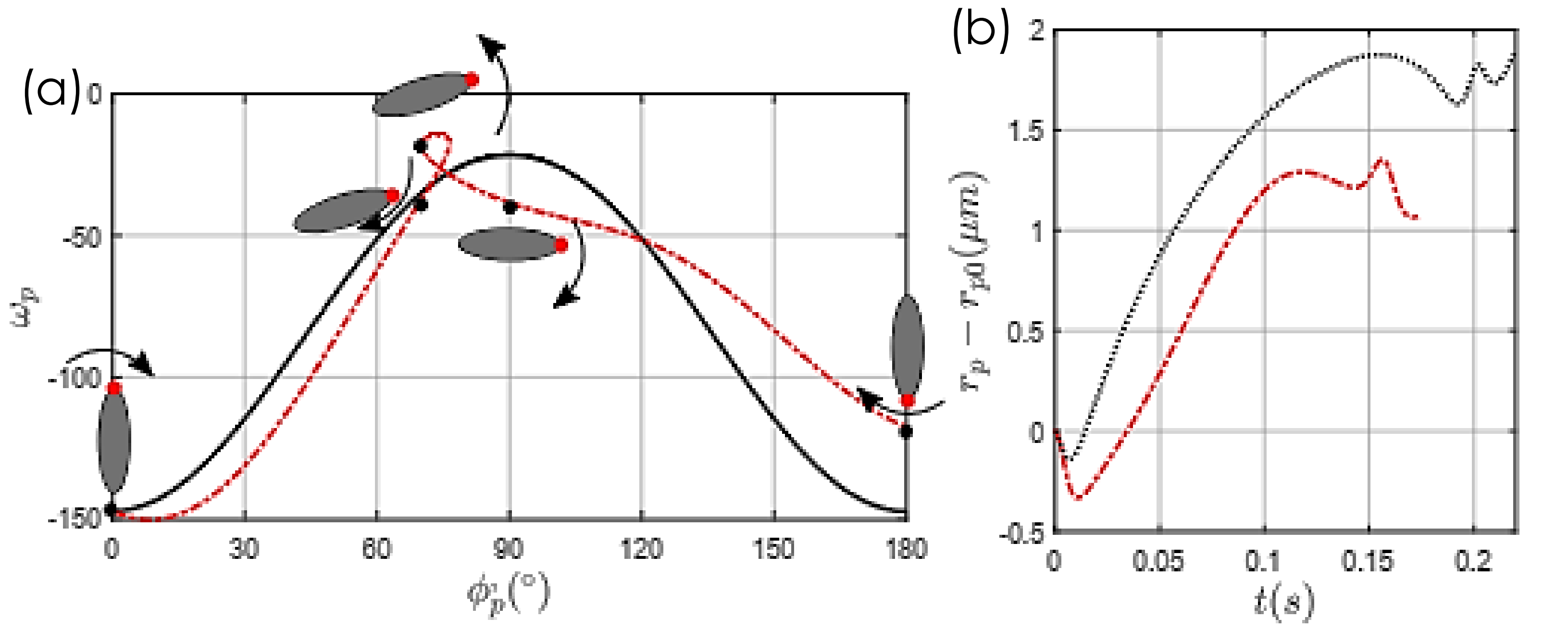

Fig2. (a) Angular velocity of an elliptical particle after one periodical rotation for H_0=0 A/m (solid black line) and for α=0° and H_0=3000 A/m (dot-dash red line). (b) Change in radial distance for an elliptical particle in a straight channel (dot black line) and in a curved channel (dot-dash red line) under a uniform magnetic field applied at α=0° with strength H_0 =3000 A/m.

- After one periodic rotation under a uniform magnetic field:
  - The particle rotates backwards during transportation, i.e., the 'loop' in Fig. 2 (a).
  - A particle in a curved channel does not migrate as much as a particle in a straight channel as seen in Fig. 2 (b).

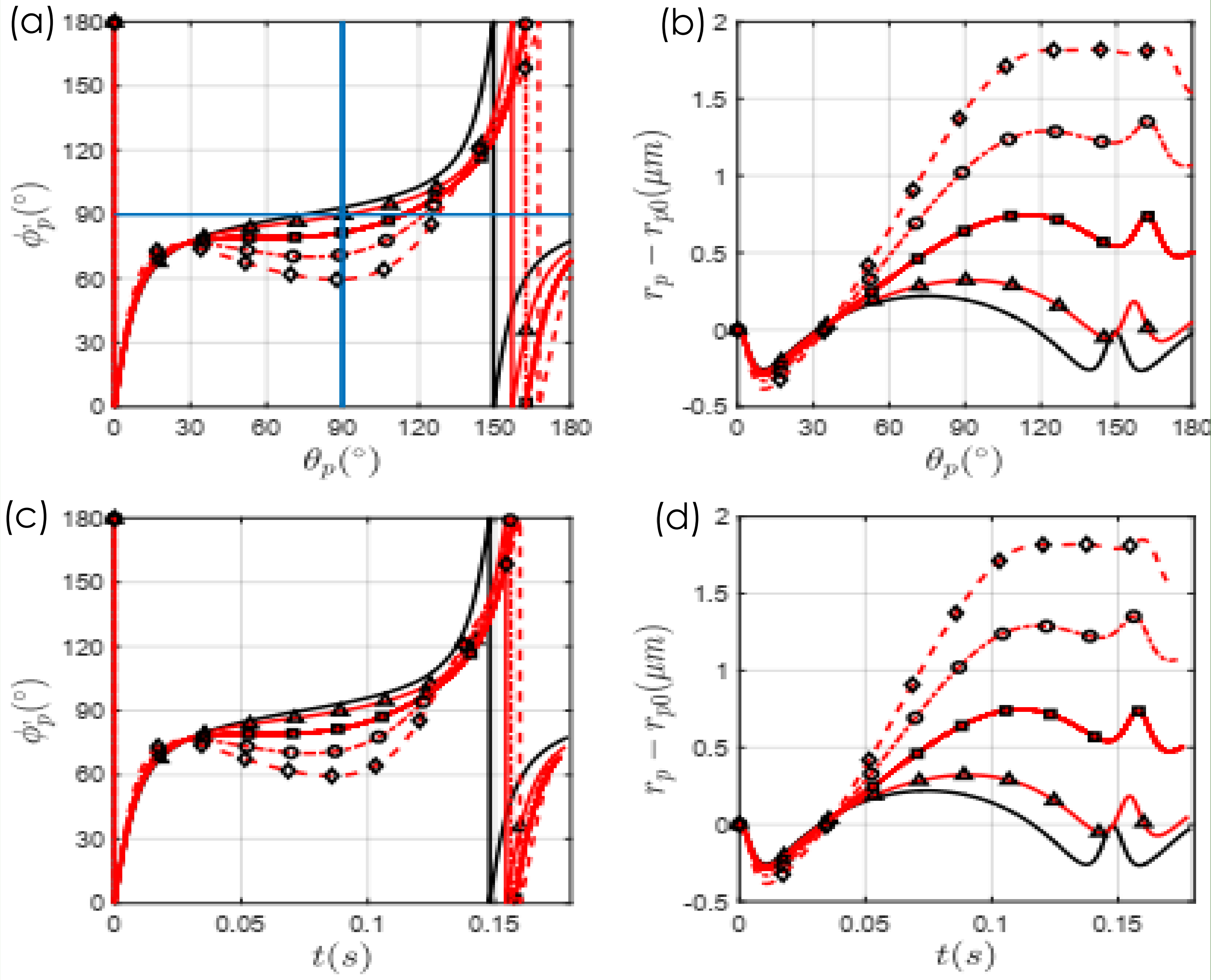

Fig3. Effect of the magnetic field strength when applied at α=0° for particle rotation with respect to θ_p (a) and time (c), respectively, and the particle's change in radial particle-wall distance with respect to (b) θ_p and time (d). The rotation and radial migration were studied for, H_0=0 A/m (solid black line), H_0=1000 A/m (triangle symbol), H_0=2000 A/m (square symbol), H_0=3000 A/m (circular symbol), and H_0=4000 A/m (diamond symbol).

- As the magnetic field strength increases, the elliptical particle migrates further away from the channel wall after one periodic rotation and as the particle exits the curved channel (θ_p = 180°).

$\alpha = 90^\circ$

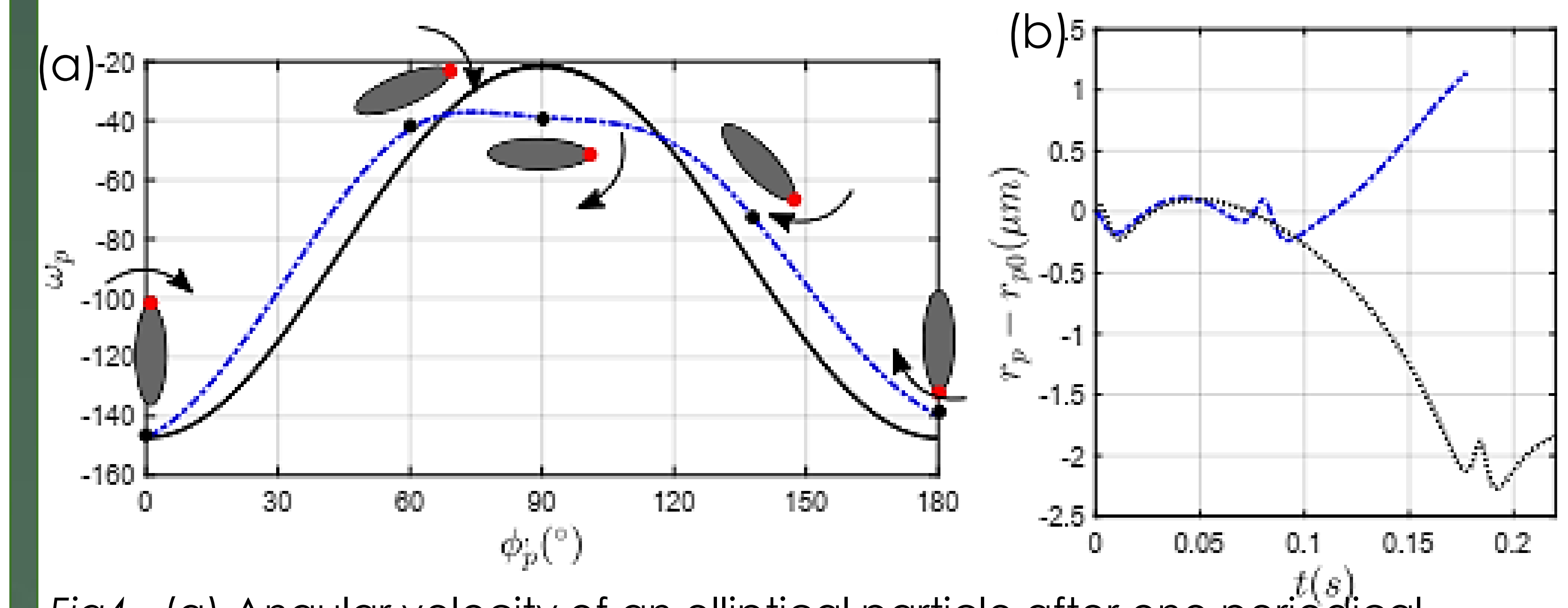

Fig4. (a) Angular velocity of an elliptical particle after one periodical rotation for H_0=0 A/m (solid black line) and for α=90° and H_0=3000 A/m (dot-dash blue line). (b) Change in radial distance for an elliptical particle in a straight channel (dot black line) and in a curved channel (dot-dash blue line) under a uniform magnetic field applied at α=90° with strength H_0 =3000 A/m.

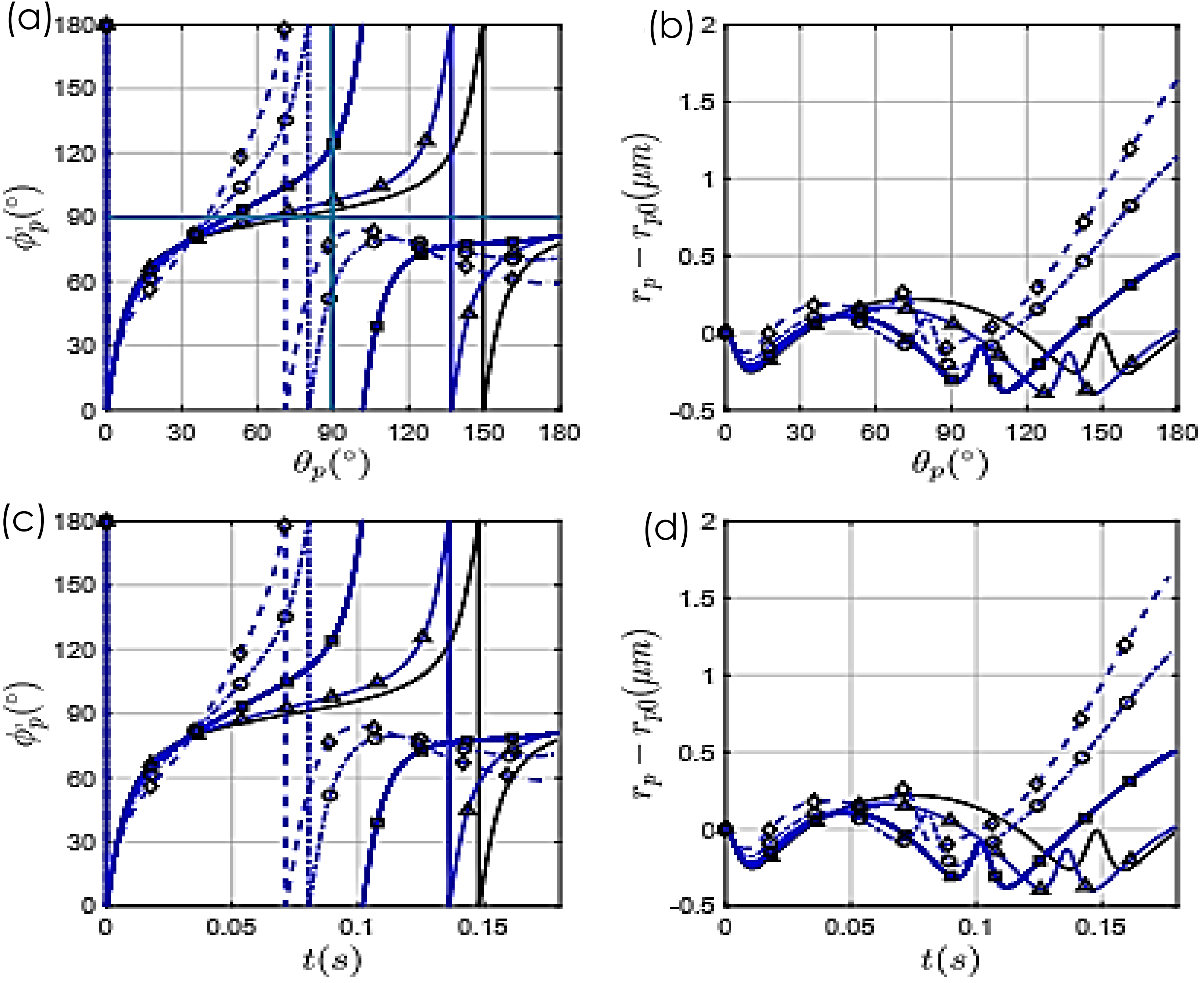

Fig5. Effect of the magnetic field strength when applied at α=90° for its rotation with respect to θ_p (a) and time (c), respectively, and the particle's change in radial particle-wall distance with respect to (b) θ_p and time (d). The rotation and radial migration were studied for, H_0=0 A/m (solid black line), H_0=1000 A/m (triangle symbol), H_0=2000 A/m (square symbol), H_0=3000 A/m (circular symbol), and H_0=4000 A/m (diamond symbol).

Conclusion

- For both magnetic field directions, as the strength increases, the elliptical particle migrates further away from the channel wall.
- In different parts of the curved channel, the particle rotates backwards because the particle's position θ_p is affected by the magnetic field differently at different directions.
- Separation of particles by shape is feasible since a spherical particle does not change its particle-wall position and we can evaluate the difference between an elliptical particle in a straight channel to an elliptical particle in a curved channel.